Written Exam Economics Summer 2020

Labour Economics

May 30, 2020 (10 AM - 10 PM)

This exam question consists of 3 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

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Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
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- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

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Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Question 1

Consider an economy, which is in steady-state with no population growth. The flow rate into unemployment is given by q_0 and the exit rate out unemployment is given by λ_0. Denote the unemployment rate for this initial steady-state equilibrium by u_0^*.

The unemployment rate at time $t = 0$ is given by $u(0) = u_0^*$. Assume that time is continuous. The economy is hit by a shock at an instant after time $t = 0$, which increases the inflow rate to unemployment to $q_1 > q_0$ and decreases the outflow rate from unemployment to $\lambda_1 < \lambda_0$. The new flow rates are permanently at, respectively, λ_1 and q_1. The new steady-state unemployment is denoted u_1^*.

1. Calculate or state the steady-state unemployment rates, u_0^* and u_1^*. Which of the two is largest?
2. Write up the change in the unemployment rate between time t and $t + dt$, where $t > 0$. Show that this gives us a differential equation if we take the limit as we let $dt \to 0$.
3. The differential equation found in the previous question is supposed to be a linear differential equation with constant coefficients of the form is $\dot{x}(t) + ax(t) = b \Leftrightarrow x(t) = Ce^{-at} + \frac{b}{a}$, where C is an unknown constant. To solve for $u(t)$, use this solution as well as the boundary conditions $u(0) = u_0^*$ and $u(t) \to u_1^*$ as $t \to \infty$.
4. Calculate how long time it takes before half of the adjustment is made (expressed by exogenous parameter values). What is the impact of λ_1 and q_1 on the convergence? Does the convergence rate speed up or down as we appoach the new steady-state?
5. Is there a theoretical model (possibly slightly altered) in the course syllabus, where a shock to the exogenous match productivity parameter can result in the described adjustment in the unemployment rate following a new and permanent higher inflow rate to unemployment and a new and permanent lower outflow rate from unemployment? If you do not think, this is the case, identify a model, which at least can give rise to part of the described dynamics and discuss why this model does not give rise to the full dynamics described above.
6. Consider the effect of the shock to the exogenous match productivity parameter in the model you have selected. Write up the equilibrium equations and explain how this shock affects all the endogenous variables. Provide economic intuition.

Question 2

Consider an economy inhabitated by N homogeneous workers. Workers search for new employment opportunities off and on the job. For simplicity, we assume that the search intensity is fixed and identical for unemployed and employed workers. The number of matches created is given by $M(U + E, V)$, where U is the number of unemployed workers, E is the number of employed workers, and V is the number of vacancies. The number of matches is increasing in both its arguments and it is assumed that the matching function has constant returns to scale. The probability of filling a vacancy is given by $\frac{M(U+E,V)}{V} = M(\frac{U+E}{V}, 1)$. Define $u \equiv \frac{U}{N}$, $e \equiv \frac{E}{N}$, $v \equiv \frac{V}{N}$, and $e + u = 1$. Using these definitions, we can write $M(\frac{U+E}{V}, 1) = M(\frac{u+e}{v}, 1) = M(\frac{1}{v}, 1) \equiv m(v)$. The probability of filling a vacancy is decreasing in the vacancy rate, i.e. $\frac{\partial m(v)}{\partial v} < 0$. The job finding rate $vm(v)$ is increasing in v, i.e. $\frac{\partial vm(v)}{\partial v} > 0$. Time is assumed to be continuous.

We assume that all firms are equally productive. Wages are set by wage posting such that a firm commits to a wage level when creating the vacancy. Therefore, let $\Pi_v(w)$ denote the discounted expected future profits of a vacancy with the promise of paying the (flow) wage w. The discounted expected future profits of a filled job at the wage w is given by $\Pi_e(w)$. Let the Bellman equation for a vacancy be

$$r\Pi_v(w) = -h + m(v)[u + (1 - u)G(w)](\Pi_e(w) - \Pi_v(w)) \tag{1}$$

where r is the discount rate, h is the flow costs of having a vacancy and $G(w)$ is the equilibrium wage distribution. The Bellman equation for a filled job is given by

$$r\Pi_e(w) = y - w + (q + vm(v)[1 - H(w)])(\Pi_v(w) - \Pi_e(w)) \tag{2}$$

where y is the productivity of the match, q is the exogenous job destruction rate, and $H(w)$ is the wage offer distribution. Free-entry in vacancy creation is assumed. This implies that $\Pi_v(w) = \Pi_v = 0$ for all w on the support of $H(w)$.

The expected present value of future incomes for an unemployed is denoted V_u, while the value of being employed at the wage w is given by $V_e(w)$. The Bellman equation for an unemployed worker is given by

$$rV_u = z + vm(v) \int_x^{\bar{w}} (V_e(\tilde{w}) - V_u) dH(\tilde{w}) \tag{3}$$

where z is the flow income as unemployed, x is the reservation wage, and $\bar{w}$ is the maximum wage. Notice also that $\tilde{w}$ is just a variable, we integrate over. The Bellman equation for an employed workers is

$$rV_e(w) = w + q(V_u - V_e(w)) + vm(v) \int_w^{\bar{w}} (V_e(\tilde{w}) - V_e(w)) dH(\tilde{w}) \tag{4}$$

1. What is the lowest wage posted in this economy? Provide economic intuition.
2. Use the Bellman equation for a vacancy and the Bellman equation for a filled job to derive the labor demand equation (or sometimes also labelled the vacancy supply equation). Provide an interpretation of this equation.
3. Derive the equilibrium wage distribution, $G(w)$, expressed in terms of the wage offer distribution, the flow rates and exogenous parameters. Interpret your result.

Now, suppose that there exist high productivity and low productivity workers. High productivity workers have the productivity y_H and low productivity workers have the productivity $y_L < y_H$. Before hiring the worker, it is not possible for employers to tell if an employee is a high productivity or a low productivity worker and the employer has to commit to the wage offer, which cannot be conditioned on worker productivity. To simplify the framework, we also assume that employers cannot terminate job matches. Hence, job matches are only terminated with the exogenous job destruction rate, q.

There are also two observable demographic groups of workers. In demographic group 1, the share of high productivity individuals is given by γ_1, whereas in demographic group 2 the share of high productivity individuals is $\gamma_2 < \gamma_1$. The expected productivity of a worker from demographic group j is $y_j = \gamma_j y_H + (1 - \gamma_j) y_L$ with $j = \{1, 2\}$ and $y_1 > y_2$.

Assume that firms can direct their recruitment to each demographic group and condition their wage offer on the demographic group. This means that you can think of the each demographic group as living on their own island. This also implies that matching frictions only exist within island and not across islands.

On each island, the economy is characterized by equations (1)-(4), besides that y should be replaced by y_1 for demographic group 1 and by y_2 for demographic group 2. Furthermore, the endogenous variables are also specific to the demographic group. Therefore, let v_j, u_j, $H_j(w)$, $G_j(w)$, $\bar{w}_j$, $\Pi_{v,j}(w)$, $\Pi_{e,j}(w)$, $V_{u,j}$, and $V_{e,j}(w)$ be specific to demographic group j.

It can be shown that the wage offer distribution, $H_j(w)$, is given by (when we assume that $r = 0$)

$$H_j(w) = \frac{q + v_j m(v_j)}{v_j m(v_j)} \left[1 - \sqrt{\frac{y_j - w}{y_j - \underline{w}_j}} \right] \quad \text{where } j = \{1, 2\} \tag{5}$$

where $\underline{w}_j$ is the lowest wage posted to workers from demographic group j.

4. Is the unemployment rate the same for the two demographic groups? Provide economic intuition. [Hint: The first step is to find out on which island the vacancy rate, v_j, is highest. You may also find it useful that $H_j(\underline{w}_j) = G_j(\underline{w}_j) = 0$.]
5. Which of the two demographic groups are offered the highest wages, on average? [Hint: Consider this by examining if one wage offer distribution stochastically dominates the other wage offer distribution using equation (5).]
6. Which of the two demographic groups is, on average, earning the highest wages in equilibrium?